From Children's Thinking to Curriculum to Professional Development to Scale: Research Impacting Early Maths Practice

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Most authors and publishers claim that their curricula or approaches to professional development are based on research, but few provide justifications. We address weaknesses in the field by presenting a model that is based on coordinated interdisciplinary research ranging from researching children's thinking and learning, to the building of curricula and professional development, to the challenges of scale-up, showing the benefit of a shared core of research-validated learning trajectories. We argue that the important results are not only increases in children's competencies, but also the sustainability of the professional development, showing an increasing effect on teaching even six years after the project's end.

Most authors and publishers claim that their curricula or professional development approaches are based on research, but few explicate their claims. We briefly assess the state of affairs regarding "research-based" curricula and practices, and then address weaknesses in the field by presenting a model that is based on coordinated interdisciplinary research ranging from mathematics research and cognitive science on children's thinking and learning to the building of curricula and professional development to the challenges of scale-up (for additional information, see Sarama & Clements, 2019).

Too often, claims of a research basis are vacuous, citing theories or empirical results vaguely (Clements, 2007, 2008; Clements & Sarama, 2013; Kinzie, Whittaker, McGuire, Lee, & Virginia, 2015). For example, authors often cite research evidence relevant to the beginning or end of the development process. That is, at the beginning, "research-based" often indicates asserting that the curriculum was built upon broad theoretical frameworks or, with little specificity, "research on children's thinking". Such a research-to-practice model alone is inadequate, because it includes a one-way translation of research results to principles to instructional designs and therefore is often flawed in its presumptions, insensitive to changing goals in the content area, and, unable to contribute to a revision of the theory and knowledge on which it is built (Clements, 2007). At the other end, research-validated may mean that effectiveness of the finished product was evaluated. This is important, but it alone leaves out critical stages of a scientific research-and-development process (Battista & Clements, 2000; Clements & Battista, 2000; Doabler et al., 2014). Further, the research designs are often weak (Munter, Cobb, & Shekell, 2016). In the area of early mathematics, for example, of 78 elementary school programs evaluated, less than 10% had valid evidence of effectiveness and four of those had only "potentially positive" effects on achievement (Doabler et al., 2014).

This is not to say that there have been no viable attempts to build valid research-based approaches–there are many (see Clements, 2008; Day-Hess & Clements, 2017; Fuson, 2009/2018; Van den Heuvel-Panhuizen, 1990). However, they remain relatively small in number and frequently do not explicate the methods and findings of the development process. To address these weaknesses, close the gap between research and practice, and increase the impact of research on the field (Cai et al., 2017), we need scientific approaches to the conceptualisation, design, creation, implementation, and scale-up of curricula that are not just "based on" or "validated by" research but that were constructed, refined, and

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evaluated with a comprehensive program of research and development (Clements, 2007, 2008; Clements & Sarama, 2013).

The Curriculum Research Framework (CRF)

Based on a review of research and expert practice (Clements, 2008), we constructed and tested a framework for the construct of research-based curricula. The goal was to promote a valid scientific curriculum development program that addresses two basic questions—about effects and conditions—in three domains: practice, policy, and theory. For example, a curriculum development program should address not only the practical question of whether the curriculum is effective in helping children achieve specific learning goals, but also the conditions under which it is effective. Theoretically, the research program should also address why it is effective and why certain sets of conditions decrease or increase the curriculum's effectiveness.

We developed the *Curriculum Research Framework* (CRF, Clements, 2007), which identifies three categories and ten phases of research-and-development, along with methods appropriate for each. A core feature of the CRF is that it is grounded in coordinated interdisciplinary research ranging from cognitive science to early childhood and mathematics education to implementation science to scale-up (the final scale-up phrase is complex and has its own elaborated model, Sarama & Clements, 2013).

Each phase must yield positive results to proceed to the next. This process can reveal weaknesses that have to be addressed and re-evaluated (or the project halted, saving resources before large-scale evaluations are conducted, too often yielding little benefit). This approach has higher validity than others for the same reason: Construct validity tests are more frequent and more trustworthy. For example, if research on children's thinking and learning in the goal domain is not carefully reviewed or conducted, it is considerably less likely that later phases of development (curricula, professional development, implementation, etc.) will be successful.

The CRF and Early Mathematics

We first implemented the CRF in the field of early mathematics education due to the low use of mathematics curricula in the earliest years of schooling in the U.S. For example, U.S. teachers tend to use emerging curricula, whereas those in China use mathematics-specific curricula (Li, Chi, DeBey, & Baroody, 2015).

The CRF Enacted

As stated, the CRF includes ten phases for asserting that a curriculum is based on research, which can be ordered by the chronology of typical curriculum development, although they are cyclic or recursive (Clements, 2007, 2008; Clements & Sarama, 2013). In the remainder of this section, we briefly describe each phase and then illustrate how we enacted that phase in the *Building Blocks* research-and-development project, an NSF-funded early childhood mathematics research and curriculum development project that was the first to be based on the CRF.

Category I: A Priori Foundations

The first category is that of *a priori foundations*. Here, the nature of the phase is a focused version of the research-to-practice model. That is, extant research is reviewed and implications for the nascent development effort drawn. The questions asked regard what is already known that can be applied in the fields such as psychology, education, and systemic change *in general* (phase 1), the *specific subject matter content* (phase 2), and *pedagogy* (phase 3).

A general guideline across all phases is to address equity issues throughout the process (Aguirre et al., 2017; Celedòn-Pattichis et al., 2018; Confrey & Lachance, 2000). For example, considerable thought should be given to the children who are envisioned as users and who participate in field tests - a convenience sample is often inappropriate. Systemic sociocultural issues should be considered as well (Tate, 1997). For *Building Blocks*, we used research on and conducted all field tests with two populations: Children from low-resource communities and children with special needs.

Phase 1. General a priori foundation.

Developers review broad philosophies, theories, and empirical results on learning and teaching. Based on theory and research on early childhood learning and teaching (Clements & Sarama, 2007), we determined that Building Blocks' basic approach would be finding the mathematics in, and developing mathematics from, children's activity. That is, we wanted to "mathematise" everyday activities, such as puzzles, songs, moving, and building–representing and elaborating mathematically.

Phase 2. Subject matter a priori foundation.

Developers review research and consult with experts to identify topics that make a substantive contribution to children's mathematical development, are generative in children's development of future mathematical understanding, and are interesting to children.

Phase 3. Pedagogical a priori foundation.

Developers review empirical findings on making activities educationally effective—motivating and efficacious—to create general guidelines for the generation of activities.

Category II: Learning Model and Learning Trajectory

Within the second category is the most extensive and intensive development phase, in which developers' structure activities in accordance with theoretically- and empirically-based models of children's thinking. This phase involves the creation of research-based learning trajectories.

Phase 4. Structure according to specific learning model and learning trajectory.

The question is how the curriculum can be constructed to be consistent with, and build upon, children's thinking and learning, which are posited to have characteristics and developmental courses that are not arbitrary, and therefore not equally amenable to various instructional approaches or curricular routes (this is based on our overarching theory of hierarchic interactionalism, to which space constraints allow only short references, but see Sarama & Clements, 2009). What distinguishes phase 4 from phase 3, which concerns pedagogical *a prior* foundation, is not only the focus on the child's learning, rather than teaching strategies alone, but also on the iterative nature of its application and the tight connections between the components of a *learning trajectory*. That is, we "conceptualise learning trajectories as descriptions of children's thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesised to move children through a developmental progression of levels of thinking, created with the intent of supporting children's achievement of specific goals in that mathematical domain" (Clements & Sarama, 2004, p. 83). In other words, each learning trajectory has three components: (a) a goal, (b) a developmental progression, and (c) instructional activities. To attain a certain mathematical competence in a given topic or domain (the goal), students learn each successive level (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects that enable thinking at each higher level (Clements & Sarama, 2014b). Research contributes to each of these components.

Mathematical progressions contribute to the identification of goals. We posit that worthwhile goals are based on the big ideas of mathematics: those that are mathematically central and coherent, consistent with children's thinking, and generative of future learning (i.e., they are part of a coherent mathematical progression, Clements & Conference Working Group, 2004; vanMarle et al., 2018).

Psychologically-oriented research, from mathematics education as well as the cognitive sciences, is critical for the creation of the developmental progression of the learning trajectories (Sarama & Clements, 2009). This is closely related to, and in part beholden to, the concepts of "growth points" (Clarke, 2008). They are synthesised from thousands of studies from interdisciplinary work around the world (Sarama & Clements, 2009).

Educationally-oriented research contributes to the third component of learning trajectories, the provision of instructional experiences (Clements & Sarama, 2014a). Here too, our learning trajectories were less a creation of all new instructional activities, and more a synthesis of the best the literature offered (e.g., DeVries, 2002; Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Griffin, Case, & Capodilupo, 1995; Kamii, 1985; Mulligan, Mitchelmore, & Prescott, 2006; Schultz, Colarusso, & Strawderman, 1989; Thomas & Ward, 2001; Van den Heuvel-Panhuizen, 1990; Wright, Martland, Stafford, & Stanger, 2002).

Category III: Evaluation

The third category, evaluation, includes phases in which empirical evidence is collected to evaluate the learning trajectories and the synthesis of them into the complete curriculum. The objective is to evaluate the appeal, usability, and effectiveness of instantiations of the curriculum. Phase 5 focuses on questions of marketability. Phases 6 to 8 involve formative research, asking whether the curriculum is usable by, and effective with, teachers and children in expanding social contexts (with teachers familiar, and then new, to the materials), and, especially, how the curriculum can be improved.

Phase 5. Market research.

Market research is usually considered commercially-oriented research about the customer, what the customer wants, and what they will buy. Typically, prototype materials are presented to "focus groups". Publishers' names and the results are hidden. Instead, we reveal all interviews and surveys we give to teachers, including their reactions to specific learning trajectories, especially the activities. The following three phases are also types of formative evaluation. In contrast to market research, these phases often involve repeated test-and-revise cycles.

Phase 6. Formative research: small group.

Pilot testing with individuals or small groups of children is be conducted on curricular components (e.g., a particular activity or software environment) or on sequences of contiguous levels. Early interpretive work evaluates components using a mix of model (or hypothesis) testing and model generation strategies, including clinical interviews, teaching experiences, and microethnographic approaches. The objective is to understand the meaning that children give to the curriculum objects and activities.

Evaluation focuses on consonance between the actions of the children and the learning model. If there are discrepancies, either the mental model, or the way in which this model is instantiated in the curriculum, they should be altered. In all cases, children's actions-on-objects enactments of their cognitive operations in the way the model posits are the focus.

Often this is the most iterative research-design phase; sometimes evaluation and redesign may cycle in quick succession, within a week to prepare modifications for another classroom, and sometimes as much as every twenty-four hours. Activities may be completed reconstituted, with edited or newly-created ones tried the next day.

Phase 7. Formative research: single classroom.

Although teachers are ideally involved in all phases, a special emphasis here is the process of curricular enactment. For example, a goal of the curriculum may be to help teachers interpret children's thinking about the activities and the content they are designed to teach; support teachers' learning of that content, especially which is probably new to teachers; and provide guidance regarding the external representation of content that the materials use. So, classroom-based teaching experiments help track and evaluate children's learning, with the goal of making sense of the curricular activities as they are experienced by individual children. At the same time, the class is observed for information concerning the usability and effectiveness of the curriculum. Ethnographic participant observation is used heavily because we wish to research the teacher and children as they interact. Thus, the focus is on how the materials are used and how the teacher guides children through the activities. This phase often involves teachers working closely with the developers. That is, the class may be taught either by a team including one of the developers and the teacher, or by a teacher familiar with and intensively involved in curricula development. The goal is to examine learning in the context of the curriculum with teachers who can enact it with a high fidelity of implementation, as opposed to ascertaining how the curriculum works in classrooms in general, which is one focus of the next phase.

Phase 8. Formative research: multiple classrooms.

In several classrooms, the entire class is observed for information about the effectiveness and usability of the curriculum, but more emphasis is placed on the usability by such teachers. Innovative materials often provide less support for teachers than the traditional materials with which they are familiar (Burkhardt, Fraser, & Ridgway, 1990), so this phase is especially important for curricula that are different than what teachers are used to. The goal of this phase is to ascertain if the supporting materials are flexible enough to support multiple situations, various modes of instruction, and different modes and styles of management.

The next two phases involve summative research, with the goal of evaluating the effectiveness (e.g., in affecting teaching practices and ultimately child learning) of the curriculum, now in its complete form, as it is implemented in realistic contexts. These two phases (9 and 10) differ from each other most markedly on the characteristic of scale.

Phase 9. Summative research: small scale.

This phase evaluates what can actually be achieved with typical teachers under realistic circumstances (Burkhardt et al., 1990), and may overlap in practice with phase 8. In multiple classrooms (2 to about 10), pre- and post-test (standardised instruments), experimental or quasi-experimental designs using measures of learning are often used, in conjunction with, and to complement, methodologies previously described.

Phase 10. Summative research: large scale.

With any curriculum, but especially one that differs from the familiar for teachers, evaluations must be conducted on a large scale (after considering issues of ethics and practical consequences, see Lester & Wiliam, 2002). Such research should use an embedded mixed methods design with a broad set of instruments to assess the impact of the implementation on participating children, teachers, program administrators, and parents, as well as document the fidelity of the implementation of the curriculum across diverse contexts. The objective should be to measure and analyse the critical variables, including contextual variables (e.g., settings, such as urban/suburban/rural; type of program; class size; teacher characteristics; child/family characteristics) and implementation variables (e.g., engagement in professional development opportunities; fidelity of implementation; leadership, such as principal leadership, as well as support and availability of resources, funds, and time; peer relations at the school; "convergent perspectives" of the developers, school administrators, and teachers in a cohort; and incentives used). A randomised experiment might be used to provide an assessment of the average impact of being exposed to the curriculum with embedded qualitative analyses.

From an integration of research on successful projects and scale-up theory and efforts, we created a model (TRIAD, for Technology-enhanced, Research-based Instruction, Assessment, and professional Development) and have evaluated it extensively with positive results, although effects decline longitudinally (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Sarama & Clements, 2013).

The TRIAD model holds that professional development should be ongoing, intentional, reflective, focused on mathematics content knowledge and children's thinking and on learning trajectories, grounded in particular curriculum materials, situated in the classroom and the school (also a synthesis of research, e.g., Bobis et al., 2005; Clarke, 1994). To realise this, we conducted both repeated (e.g., > 10) full-day sessions of training in regular meetings and frequent coaching. Training included all three components of each learning trajectory, the goal, the developmental progression, and the instructional activities and strategies. To understand the goal, teachers study the mathematical content. A key instructional use of learning trajectories is in formative assessment along the developmental progression. We worked with teachers to study a developmental progression, analyse multiple video segments illustrating each level and discuss the mental "actions on objects" that constitute the defining cognitive components of each level; order tasks corresponding to those levels; and practise diagnosis in teams, with a couple of teachers exemplifying behaviours of children at different levels, and one teacher identifying the level of each (we used an online application; an update to it can be seen at learningtrajectories.org). Further, teachers need training in understanding, administering, and especially using data from new assessment strategies (Foorman, Santi, & Berger, 2007). TRIAD training focuses mainly on the curriculum-embedded assessment.

Formative assessment requires more than identifying children's levels of thinking. Teachers must select and modify instructional activities and strategies that are appropriate and effective for each level. To learn about instructional tasks and strategies, teachers practice the curriculum's activities, but also analyse them to establish and justify their connection to a particular level of the developmental progression.

Across all forms of professional development, the focus is on children's thinking and learning. Conversations about children's learning serve as way to address implementation issues. Although early mathematics is often an uncomfortable topic for early childhood educators, the newness of the learning trajectories for all participants helps establish a sense of shared learning and community. Each session in the last third of professional development includes scheduled time to discuss "learning stories" (Perry, Dockett, & Harley, 2007). Teachers show their record keeping on small group record sheets, and sometimes videos, and discuss their use of learning trajectories in teaching children, including challenges, questions, and successes. These discussions promote peer learning and collaboration and also motivate peers to solve implementation difficulties.

Evaluation of The CRF and TRIAD

Multiple evaluations of TRIAD support the efficacy of our CRF. Briefly, in a cluster randomised trial study of effectiveness, 42 schools serving low-resource communities were randomly assigned to three treatment groups using a randomised block design involving 1375 pre-schoolers in 106 classrooms. Teachers implemented the intervention with adequate fidelity. Pre-schoolers in the TRIAD groups learned more mathematics than those in the control group (effect size, g = 0.72) (Clements et al., 2011).

There were two TRIAD groups. Identical in pre-K, one included follow through in the kindergarten and first grade years, including knowledge of the pre-K intervention and ways to build upon that knowledge using learning trajectories. Students in this follow-through group scored significantly higher than control students ($g = 0.51$ for those who received follow through intervention in kindergarten and 1st grade; $g = 0.28$ for non-follow through) and follow-through students scored significantly higher than non-follow-through students ($g = 0.24$) (Clements, Sarama, Wolfe, & Spitler, 2013).

Also important is the *sustainability* of the impact on *teachers.* Although a logical expectation would be that, after the cessation of external support and professional development provided by the research staff, teachers would show a pattern of decreasing levels of fidelity, these teachers *actually demonstrated increasing levels of fidelity*, continuing to demonstrate high levels of sustained fidelity in their implementation of the underlying curriculum two years past exposure. The strongest indicator of high fidelity was child gain (Clements, Sarama, Wolfe, & Spitler, 2015). Just as striking, teachers also demonstrated sustained levels of fidelity as long as six years after the end of the intervention. Notable is these teachers' encouragement and support for discussions of mathematics (willingness and ability to listen to students and support students' understanding as they listen to others) and their use of formative assessment (Sarama, Clements, Wolfe, & Spitler, 2016).

Implications

Children, especially those from low-resource communities, need more mathematics education in preschool (Bodovski & Farkas, 2007; Sarama & Clements, 2009). Evidence from both educational (National Research Council, 2009; Paris, Morrison, & Miller, 2006) and economic (Carneiro & Heckman) research suggests that early education is the most important period in which to invest resources.

Immediate Effects in Preschool

The present study provides additional empirical support for the hypothesis that education based on learning trajectories, as implemented here, helps teachers provide more and better mathematics for their preschoolers. Educational environments that focus on conceptual understanding and encourage students to develop, discuss, and use strategies for solving challenging problems appear to develop both "basic" competencies and higher-level processes.

The results provide empirical support for the effectiveness of mathematical learning trajectories as a base for both curriculum and teacher training that engenders shared, systematic practice. It also argues, in contrast to those who champion an individual teacher's idiosyncratic interpretation and implementation of curriculum, that such systematic practice is more effective and amenable to scientifically-based improvement than private, idiosyncratic practice (Raudenbush, 2009). This is not to say that teachers could or should implement curricula in routinised ways and certainly not that they should deliver "scripted" curriculum with little or no interpretation. Indeed, such an approach would stand in contraposition to the use of hypothetical learning trajectories in the service of formative assessment. Instead, we propound the following three related points. First, although teachers do interpret curriculum and must be sufficiently knowledgeable and competent to implement it in their classroom context, focusing on the shared scientific base and the common goals, such as developmental progressions and instructional tasks in learning trajectories, is a more effective and efficient way to improve education for children as opposed to focusing primarily on teachers' autonomous invention of individualistic curricula (Raudenbush, 2009). Second, such scientifically-grounded shared practice is, somewhat paradoxically, more likely to generate creative contributions. This is so because they will constitute modifications of effective practice that is already shared, and thus understood and more easily adopted, and that in turn will be accessible to discussion and further scientific investigation.

The Need for Follow Through

The results from kindergarten and first grade provides compelling evidence regarding the importance of follow through. With such follow through, the effects from the pre-K intervention persisted; without follow through, they were significantly smaller. Multiple researchers have reported that preschool benefits do not persist; that is, that gains "fade" (Leak et al., 2012; Natriello, McDill, & Pallas, 1990; Preschool Curriculum Evaluation Research Consortium, 2008; Turner, Ritter, Robertson, & Featherston, 2006)—a main rationale for the follow-through component and this study. We believe that such an interpretation mistakenly treats initial effects of interventions as independent of the students' future school contexts. That is, these researchers theoretically reify the treatment effect as an entity that should persist unless it is "weak" and thus susceptible to fading. Such a perspective identifies the gain as a static object carried by the student that, if not evanescent, would continue to lift the student's achievement above the norm. Our theoretical position and this study's empirical results support an alternative view. Successful interventions do provide students with new concepts, skills, and dispositions that change the trajectory of the students' educational course. However, these are, by definition, exceptions to the normal course for these students in their schools. Because the new trajectories are exceptions, multiple processes may vitiate their positive effects, such as institutionalisation of programs that assume low levels of mathematical knowledge and focus on lower-level skills and cultures of low expectations for certain groups (and, as noted, kindergarten and first grade instruction often covers material children already know even without pre-K experience, Engel, Claessens, & Finch, 2013; Van den Heuvel-Panhuizen, 1996). Left without continual, progressive support, children's nascent learning trajectories revert to their original, limited course.

Professional Development

Learning trajectory-based instruction engaged in by the pre-kindergarten teachers in this study may have provided a coherent program of teaching and learning, which promoted the significant levels of fidelity found in this study. Teachers taught the curriculum with increasing fidelity as time went by, even though research project staff was no longer able to provide support. They seemed to have internalised the program (Timperley, Wilson, Barrar, & Fung, 2007) and to have made sense of the curricular activities involved with whole group, small group, and other components, within an overall structure of learning trajectories that progressed toward a known mathematical goal. By engaging in the initial professional development, and then, becoming empowered by their own knowledge of the trajectories and the ways to support learners through the trajectories, as well as by their perceptions that their students were learning more, they became progressively more faithful to the intended program, instead of drifting from it as time elapsed and support disappeared, as has been documented by other studies (Datnow, 2005; Hargreaves, 2002). One implication, then, is that a coherent model of professional development, curriculum, instruction, and assessment based on learning trajectories may provide the conditions for promoting sustainability in teacher practices (Clements & Sarama, 2014b), and may be particularly beneficial in addressing the climate of low expectations in urban schools (Johnson & Fargo, 2010), as teachers increase their understanding of the capacities of all children to learn mathematics.

As teachers come to understand children's probable developmental paths and become adept at anticipating children's strategies and misconceptions, their teaching practices may become more grounded and solidified. As they monitor for children's multiple models, and probe for the ways in which children's mathematical thinking fits the structure of the trajectory, their teaching practices may become reinforced as student reactions provide positive feedback for their practices (Guskey, 2002). Teachers who demonstrate sustained fidelity of implementation to a program that has demonstrated improved child achievement will have a positive impact on many more children than teachers who implement with fidelity only during treatment. Thus, another general implication is that helping teachers perceive and document their children's learning (which human subjects constraints disallowed in this research) may be an effective way to maintain and even increase fidelity of implementation. These positive perceptions of learning may be especially important in motivating teachers to productively face the challenges inherent in fully implementing all aspects of the curriculum, which bring their own difficulties. For example, educational technology challenges include limited hardware, hardware and software problems and limited troubleshooting competencies, difficulty scheduling computer use for all children, and inconsistency between computer use and customary practice including contextually constrained choices (Clements & Sarama, 2008; Cuban, 2001). Solving problems successfully, such as engaging children productively in technology activities, may engender confidence and risk-taking in future work. Simply, success breeds success, and such changes in practice may lead to positive changes in beliefs (e.g., Showers, Joyce, & Bennett, 1987), again, resulting in co-mutually reinforcing changes in beliefs and practices (Caudle & Moran, 2012) rather than conflicts between them.

Final Words

The TRIAD model is not simply about a new curriculum or training teachers to use it. Success required complex changes, including a change in instructional structures, pedagogical strategies, and classroom communication and culture (Grubb, 2008). Given the importance of early competence in mathematics (e.g., Duncan et al., 2007; Paris, Morrison, & Miller, 2006), the TRIAD implementation described here has implications for practice and policy, as well as research. TRIAD's guidelines should be considered when planning to increase the quality and quantity of early mathematics education.

Future Directions

Future research and development might evaluate the CRF's implementation with other grade levels and other topics (see Doabler et al., 2014; Kinzie et al., 2015). Just as importantly, our research designs could not identify which components of the CRF and TRIAD models and their instantiations are critical. Such research would be theoretically and practically useful. The specific contribution of the learning trajectories *per se* needs to be disentangled and identified. In a present project, we are addressing this issue. In an IES-funded project entitled, "Evaluating the Efficacy of Learning Trajectories in Early Mathematics", we are testing the efficacy of learning trajectories in a series of eight randomised clinical trials testing different aspects of LTs. These experiments will determine whether LTs are more efficacious than other approaches in supporting young children's learning.

On a practical side, with funding from the Heising-Simons Foundation and the Bill and Melinda Gates Foundation, we are developing a technology-based tool for teachers and teacher trainers that extends a resource we created for the TRIAD evaluation. The Learning and Teaching with Learning Trajectories tool, or LT^2, is a new, free resource for early mathematics (see www.LearningTrajectories.org). LT^2 provides learning-trajectories-based mathematics resources for teachers, caregivers, and parents. LT^2 runs on all technological platforms, addresses new ages—birth to age 8 years—and includes new alignments with standards and assessments, as well as new software for children. from—their everyday activities, including art, stories, puzzles, and games.

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